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A reappraisal of why aspirating pipes do not flutter at infinitesimal flow

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Abstract

In an earlier Brief Note, in 1999, a simplified analysis was presented to show why aspirating pipes—a problem related to Feynman's aspirating rotary-sprinkler quandary—do not flutter at infinitesimally small flow-rates. Recently, however, it has become clear that this earlier work is at best incomplete. A reevaluation of the problem is undertaken here, with some fresh insights as to if and why flutter does not occur at low flow velocities. In the process, the equation of motion is derived by an appropriate statement of Hamilton's principle, as well as by Newtonian methods, and the work done by the fluid is computed.

1. Introduction

A brief historical perspective would be a useful preamble to the presentation of some new ideas on this problem.

Perhaps the earliest work on this topic was by Païdoussis and Luu (1985), who found that, in the absence of dissipation, an aspirating cantilevered pipe loses stability at infinitesimally low flow velocities, and then regains stability at higher flow. That is, the predicted linear dynamical behaviour is the mirror image of that of a cantilevered pipe *discharging* fluid, which is stable at low flows and then loses stability by flutter at a higher flow—in the absence of dissipation, at the very same flow velocity at which the aspirating pipe regains stability. This result is obtained essentially by changing the flow velocity U in the equation of motion of the discharging cantilever (Gregory and Païdoussis, 1966a; Païdoussis, 1970, 1998) to $-U$, thus effectively presuming a purely tangential ingestion of the fluid at the free end (a reverse-jet flow).

Subsequently, as a result of discussions with Dr D.J. Maull at Cambridge University in 1995, this problem was linked to that of Feynman's quandary on the sense of rotation of an aspirating rotary lawn-sprinkler: would it be forward (as for the normal, discharging sprinkler) or backward (Gleick, 1992)? Thence, the problem was reevaluated and a new equation of motion was proposed, on the basis of which it was concluded (Païdoussis, 1998, 1999) that "aspirating pipes do not flutter at infinitesimally small flow". The gist of the mechanism underlying this result is that the unsteady centrifugal forces over arbitrarily bent segments of the pipe are cancelled out by equal and opposite unsteady forces due to depressurization, associated with the negative mean pressure generated by the ingestion of fluid at the free end and applying all along the pipe. Physically, this implies a spherical sink flow at the pipe inlet, of effectively zero average flow

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velocity in any given direction, changing to a flow with velocity $-U$, aligned with the pipe, upon entry into the pipe itself.

An experiment was conducted (Païdoussis, 1999), involving two nominally identical vertical, cantilevered, flexible pipes, submerged in a water tank; at the free end, each pipe was fitted with a 90° elbow. One pipe discharged water at the free end, while the other aspirated. When the flow was turned on, the discharging pipe bent backwards in the plane of the elbow as a result of the centrifugal force at the elbow; on the other hand, after a starting transient, the aspirating pipe remained vertical and undeformed. This observation lent support to the theoretical conclusion regarding stability of aspirating pipes, at least at low flow-rates.

More recently, however, fresh doubts surfaced [see Kuiper and Metrikine (2005)] to the effect that (i) the depressurization effect may have been overestimated, and (ii) the system may actually lose stability by the action of Coriolis forces alone, a possibility unaccountably ignored in the Païdoussis (1998, 1999) analysis.

Accordingly, the stability of the system under consideration is here reappraised *ab initio*, adopting along the way some ideas originally put forward by Pramila (1992).

Before closing this Introduction, it ought to be recalled that this problem is not wholly academic. One important application is in ocean mining, e.g. of manganese nodules, by essentially vacuuming the sea floor from a surface vessel, thus involving a long, flexible, aspirating pipe [see, e.g., Chung et al. (1980); Deepak et al. (2001); Xia et al. (2004)].

2. Background theory

2.1. General considerations

Consider the simplest form of the linearized equation of motion of an undamped horizontal cantilevered pipe conveying fluid,

$$EI\,\frac{\partial^4 w}{\partial x^4}+MU^2\,\frac{\partial^2 w}{\partial x^2}+2MU\,\frac{\partial^2 w}{\partial x\partial t}+(M+m)\,\frac{\partial^2 w}{\partial t^2}=0, \tag{1}$$

where x and t are the axial coordinate and time, respectively, EI is the flexural rigidity of the pipe, M the mass of fluid per unit length, flowing from the fixed end ($x=0$) to the free one ($x=L$) with a steady flow velocity U, m the mass of the pipe per unit length, and w the lateral deflection of the pipe [see, e.g., Païdoussis (1998)]. Thus, for the present, we consider the pipe discharging rather than aspirating fluid. The first term in Eq. (1) is the flexural restoring force. Upon recalling that $\partial^2 w/\partial x^2 \sim 1/R$, where R is the local radius of curvature, it is obvious that the second term is associated with centrifugal forces as the fluid flows in curved portions of the pipe. Similarly, the third term is recognized as being associated with the Coriolis acceleration, and the last term represents inertial effects.

The dynamics of the system for the discharging cantilever, i.e. for $U>0$, is well understood. For sufficiently small U, the dynamics is dominated by the Coriolis force $2MU(\partial^2 w/\partial x\partial t)$, and the system is subjected to flow-induced damping. For sufficiently large U, however, the centrifugal force, $MU^2(\partial^2 w/\partial x^2)$, which may also be viewed as a compressive follower force, overcomes the Coriolis damping effect, and the system loses stability by single-mode flutter via a Hopf bifurcation.

Considering periodic motions of period T, it is shown (Benjamin, 1961a; Païdoussis, 1970, 1998) that the work done by the fluid on the pipe is equal to

$$\Delta W=-MU\int_0^T\left[\left(\frac{\partial w}{\partial t}\right)_L^2+U\left(\frac{\partial w}{\partial t}\right)_L\left(\frac{\partial w}{\partial x}\right)_L\right]\mathrm{d}t\neq 0, \tag{2}$$

where $(\partial w/\partial t)_L$ and $(\partial w/\partial x)_L$ are, respectively, the lateral velocity and slope of the free end. For small $U>0$, the first term dominates, and the work done is negative; hence, the pipe loses energy to the flowing fluid, and free pipe motions are damped. For high enough U, however, the second term dominates; if the slope and velocity of the free end have opposite signs over a period, $[(\partial w/\partial x)_L(\partial w/\partial t)_L]|_0^T<0$, then the work done may be positive, and energy may then flow from the fluid (a source of unbounded energy) to the pipe, resulting in amplified oscillations. The aforementioned opposite-sign characteristic of the free-end slope and velocity corresponds to the "dragging, lagging" form of flutter, observed in experiments and commented upon by Bourrières (1939), Benjamin (1961b) and Gregory and Païdoussis (1966b).

Consider next the situation with $U<0$, i.e. the aspirating system, presuming that Eq. (1) still holds true, with $-U$ instead of U. Exactly the *opposite* conclusions are then reached by consideration of Eq. (2): (i) in the course of free motions, the pipe absorbs energy from the fluid for sufficiently small $|U|$ and is therefore subject to flutter; (ii) for higher

$|U|$, the pipe loses energy to the fluid, and hence it is stabilized and its motions are damped. Consequently, the startling conclusion is reached that the system is unstable for infinitesimally small $|U|$—or, if dissipation is taken into account, for *quite* small $|U|$. This is precisely what was obtained via a full-fledged linear analysis of the system by Païdoussis and Luu (1985).

If these findings were true, there would be serious repercussions on the feasibility of ocean mining (Païdoussis, 1999). Hence, experimental verification would be highly desirable.

Several attempts to verify experimentally these findings failed: the pipe remained inert as the flow velocity was increased, up to the point where it collapsed as a shell in the second circumferential mode (i.e., it became flattened), close to the point of clamping (upper end of the vertically mounted, totally immersed pipe). This is the point of maximal differential pressure, due to viscous pressure drop, between the lower internal pressure and the higher pressure in the external stagnant fluid. Similar was the experience and the experimental set-up itself devised by Feynman to test the rotation of an aspirating rotary sprinkler.[1]

Clearly, therefore, the foregoing constitutes a paradox: theory predicts that the aspirating pipe loses stability at infinitesimal (or quite small) flow velocity, but experiments show the system to remain stable, at least to the maximum attainable flow prior to pipe collapse. Reversing the flow direction in the experiments does not invert the stability behaviour of the pipe. Similarly, in Feynman's sprinkler, reversing the flow direction did not reverse (nor replicate) the direction of rotation.

2.2. First revision of the theory, and state of knowledge prior to 2004

In the course of documenting this work (Païdoussis, 1998) after the aforementioned discussions with David Maull, a fresh attempt was made to resolve the paradox.

It was first realized that the pressure at the inlet of the pipe is lower than that of the ambient surrounding fluid. Considering the force on the fluid at the pipe inlet to be equal to the change in the momentum before and after entry, one may write

$$F_x = MU(\Delta U)_x \equiv MU(U_{xi} - U_{xo}), \tag{3}$$

where MU is the mass flow-rate and $(\Delta U)_x$ is the change in flow velocity as the fluid enters the pipe; U_{xo} is the average flow velocity in the axial direction just *outside* the pipe, and U_{xi} is the value *inside* the pipe. Considering a sink flow (Fig. 1) at inlet, $U_{xo} \simeq 0$; and, of course, $U_{xi} = -U$.[2] Thus, $F_x = -MU^2$. Hence, the force of the fluid on the pipe, $F_x^* = -F_x$, is given by

$$F_x^* = MU^2. \tag{4}$$

Without loss of generality, from here on gravity and buoyancy forces on the generally vertical pipe will be ignored for simplicity, as if the pipe were horizontal. An axial force balance at the free end of the pipe gives $F_x^* = (T - pA)_L$, and hence

$$(T - pA)_L \equiv (T_L - p_L A) = MU^2, \tag{5}$$

where T is the tension and p the internal pressure, measured above the external ambient pressure, both at $x = L$ [see Païdoussis, (1998, Eq. 3.98)]; the term $T - pA$ is often called the *effective tension*. Here, since $T_L = 0$ (but see also Appendix A for $T_L \neq 0$), this is viewed as a pressurization effect; thus, $p_L A = -MU^2$, indicating a *depressurization* at the inlet. Moreover, since an axial force balance along the pipe gives

$$\frac{\partial}{\partial x}(T - pA) = 0, \tag{6}$$

this depressurization applies all along the pipe. Eq. (6) means that frictional forces on the pipe, hence the x-varying tension, and frictional pressure loss cancel out (Gregory and Païdoussis, 1966a, b; Païdoussis, 1998). Consequently, the only tension which can exist is an externally applied tension, $T_L = \overline{T}$, and an externally induced pressurization, $p_L = \overline{p}$, both applicable for $0 \leqslant x \leqslant L$. In their presence, for the aspirating pipe (flow velocity $-U$), Eq. (1) is modified to

$$EI\frac{\partial^4 w}{\partial x^4} - (\overline{T} - \overline{p}A)\frac{\partial^2 w}{\partial x^2} + MU^2\frac{\partial^2 w}{\partial x^2} - 2MU\frac{\partial^2 w}{\partial x \partial t} + (M + m + M_a)\frac{\partial^2 w}{\partial t^2} = 0, \tag{7}$$

[1] Similar also were the ensuing accidents while attempting to increase the flow excessively; see Païdoussis (1998, 1999).

[2] See also Appendix A.

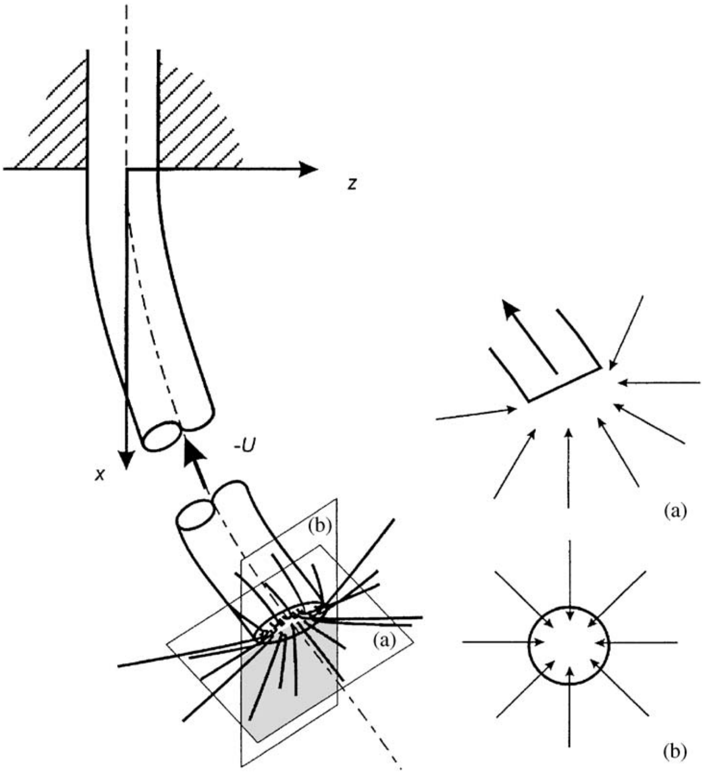

Fig. 1. Diagram showing the sink-like flow at the pipe outlet of an aspirating pipe. (a) Side-view and (b) cross-sectional view of the inlet flow.

(Païdoussis, 1998, Eq. (3.98)), in which the added mass per unit length M_a of the ambient fluid has been included. Hence, since $T_L - p_L A \equiv \overline{T} - \overline{p}A$, in view of Eq. (5) all centrifugal force terms (i.e., terms involving $\partial^2 w/\partial x^2$) in Eq. (7) cancel out!

It was because of this fact, obtained a little differently (see Appendix A), and by analogy to the discharging cantilever, that it was concluded in Païdoussis (1998, 1999) that the system is stable at infinitesimal flow velocities. As we shall see, this is at best incomplete.

Pramila (1992) also conducted some work on the problem, which was in fact more perceptive and complete (see Section 3.1), concluding that "cantilevered pipes aspirating fluid may be stable".

2.3. Lacunae and doubts

Recently, doubts were raised about the generality and correctness of some aspects of the work outlined in Section 2.2. Briefly, these may be summarized as follows:

(i) even if all centrifugal terms cancel out, the Coriolis term, which for a discharging pipe damps motions for small U, in the case of an aspirating pipe generates negative damping for flow velocities $-U$, thus causing instability, essentially as originally found by Païdoussis and Luu (1985);
(ii) the result in Eq. (5) is doubtful since it contravenes Bernoulli's equation for the flow from a stagnant state far away to the pipe inlet, as will be detailed below; hence the centrifugal forces may not cancel out!

According to Bernoulli's equation, presuming a flow not quite like a pure sink flow at the inlet, we can write

$$p_\infty + \frac{1}{2}\rho U_\infty^2 = p_o + \frac{1}{2}\rho U_o^2,$$

in which the subscript ∞ means "far away", and hence $U_\infty = 0$; the subscript o means "just facing the pipe" and, according to Kuiper and Metrikine (2005) also within the inlet. Hence, the depressurization thus obtained in

$p_L = p_o - p_\infty = -\frac{1}{2}\rho U_o^2 \equiv -\frac{1}{2}\rho U^2$; therefore, writing $\rho A = M$, this gives

$$\bar{p}A = -\frac{1}{2} M U^2, \tag{8}$$

i.e., only half as large as what was given by Païdoussis (1998, 1999); presuming $\overline{T} = 0$, then $-(\overline{T} - \bar{p}A) = -\frac{1}{2} MU^2$. Thus, Eq. (7) becomes

$$EI \frac{\partial^4 w}{\partial x^4} + \frac{1}{2} MU^2 \frac{\partial^2 w}{\partial x^2} - 2MU \frac{\partial^2 w}{\partial x \partial t} + (M + m + M_a)\frac{\partial^2 w}{\partial t^2} = 0. \tag{9}$$

On the basis of this equation, the work done by the fluid on the pipe in the course of a putative cycle of oscillation of period T is found to be

$$\Delta W = MU \int_0^T \left[\left(\frac{\partial w}{\partial t}\right)^2 - \frac{1}{2} U \frac{\partial w}{\partial x}\frac{\partial w}{\partial t} \right]_L \mathrm{d}t = 0.$$

Therefore, flutter is again predicted at infinitesimal flows, *à la* Païdoussis and Luu (1985), followed by restabilization at somewhat higher U than originally predicted (because of the $\frac{1}{2}$ factor).

Items (i) and (ii) are discussed in detail in a critique by Kuiper and Metrikine (2005). The authors believe that item (i) is absolutely correct and item (ii) has considerable merit.[3] Therefore, a reappraisal of the dynamics of the system has been initiated, as described next.

3. Reappraisal of the dynamics of aspirating pipes

3.1. The basic model

Fig. 2(a) shows the end of the pipe inclined at an angle $\chi \equiv \tan^{-1}(\partial w/\partial x)_L \simeq w'_L$, where $(\ )' = \partial(\ )/\partial x$. In addition to the (x, z)-coordinate system, we shall also use the (ξ, ζ) system. We postulate a small mean flow velocity $-v$ facing the inlet, in contrast to the work in Section 2.2 where $v = 0$; we further postulate that, for small motions, $-v$ remains in the mean direction of the pipe end, i.e. tangential to the undeflected pipe (Fig. 2(c)). Hence, the forces exerted on the fluid at the inlet, equal to the change in momentum, $MU(\Delta U)$, are

$$F_x = MU[-U \cos\chi - (-v)] = -MU(U - v), \tag{10a}$$

$$F_z = MU[(\dot{w}_L - U \sin\chi) - 0] = MU(\dot{w}_L - Uw'_L), \tag{10b}$$

correct to $\mathcal{O}(\varepsilon)$, where $w \sim \mathcal{O}(\varepsilon)$. In F_z, it is recognized that the fluid in the pipe has a velocity $\dot{w}_L$ in the z-direction, equal to the pipe-end velocity, as well as the tangential flow velocity $-U$; whereas, outside the pipe, the velocity in the z-direction is null.

In the foregoing, F_z had been wholly ignored; however, its existence and form were proposed some time ago by Pramila (1992), along with $F_x = -MU^2$—but, unfortunately, this important work remained in relative obscurity [see Païdoussis, (2003, Appendix Ω)].

Consequently, the forces of the fluid on the pipe, denoted by an asterisk, are

$$F_x^* = MU^2(1 - \alpha), \quad F_z^* = -MU(\dot{w}_L - Uw'_L), \tag{11}$$

where $\alpha = v/U$. Clearly, according to Section 2.2, $\alpha = 0$; according to Section 2.3, $\alpha = \frac{1}{2}$; finally, if the flow at inlet is equal to $-U$, as in Section 2.1, then, $\alpha = 1$. We also obtain (Fig. 2(b)), correct to $\mathcal{O}(\varepsilon)$:

$$F_\xi^* \simeq F_x^*, \quad F_\zeta^* = F_z^* \cos\chi - F_x^* \sin\chi = -MU(\dot{w}_L - \alpha U w'_L). \tag{12}$$

The equation of motion continues to be Eq. (7), but we now include a viscoelastic damping in the pipe material and linearized viscous damping due to the surrounding fluid; thus,

$$EI\left(1 + \alpha^* \frac{\partial}{\partial t}\right)\frac{\partial^4 w}{\partial x^4} + [MU^2 - (\overline{T} - \bar{p}A)]\frac{\partial^2 w}{\partial x^2} - 2MU \frac{\partial^2 w}{\partial x \partial t} + c\frac{\partial w}{\partial t} + (M + m + M_a)\frac{\partial^2 w}{\partial t^2} = 0. \tag{13a}$$

[3] In fact, Kuiper and Metrikine (2005) were sufficiently ambivalent on this point as to conduct calculations in their analysis with $\alpha = 0$, $\frac{1}{2}$ and 1; α is defined in Eq. (11).

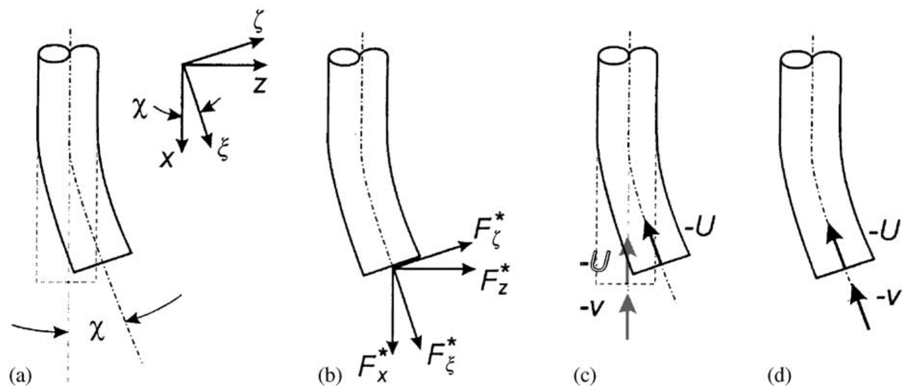

Fig. 2. (a) The free-end of the pipe and definition of the coordinate systems used and the angle χ; (b) definition of the forces exerted by the fluid on the pipe. (c) The inlet flow assumed in Section 3.1, with v always in the direction shown. (d) The inlet flow assumed in Section 3.2, with v tangential to the free end of the pipe.

At the free end ($x = L$), the bending moment is zero, and the shear force is related to F_ζ^*; i.e.,

$$EI\,\frac{\partial^3 w}{\partial x^3} - MU\left(\frac{\partial w}{\partial t} - \alpha U\,\frac{\partial w}{\partial x}\right) = 0. \tag{13b}$$

This latter may be inserted in the equation of motion by means of a Dirac delta function [see, e.g., Païdoussis, (1998, Section 2.1.3)], and since $\overline{T} - \overline{p}A = F_\xi^*$ as for Eq. (5), with the aid of Eqs. (11) and (12), we have

$$EI\left(1 + \alpha^*\frac{\partial}{\partial t}\right)\frac{\partial^4 w}{\partial x^4} + \alpha M U^2\frac{\partial^2 w}{\partial x^2} - 2MU\,\frac{\partial^2 w}{\partial x \partial t} + c\,\frac{\partial w}{\partial t} + (M + m + M_a)\frac{\partial^2 w}{\partial t^2} + MU\left(\frac{\partial w}{\partial t} - \alpha U\,\frac{\partial w}{\partial x}\right)\delta(x - L) = 0. \tag{14}$$

The same equation has been obtained via Hamilton's principle; this is not given here for brevity, but will be in a forthcoming full-length paper.

In the absence of dissipation, it can easily be shown that the work done on the pipe by the fluid over a cycle of oscillation is $\Delta W = 0$ for all values of U, irrespective of what the value of α is; even if $\alpha = 1$. Hence, according to this model, *with* damping the system is unconditionally stable.

In addition to energy transfer considerations, stability was also assessed from the eigenfrequencies of the system. These were computed via a two-mode Galerkin solution of the dimensionless form of Eq. (14) obtained by using

$$\eta = \frac{w}{L}, \quad \xi = \frac{x}{L}, \quad \tau = \left(\frac{EI}{M + m + M_a}\right)^{1/2}\frac{t}{L^2}, \quad \beta = \frac{M}{M + m + M_a},$$
$$\sigma = \frac{cL^2}{[EI(M + m + M_a)]^{1/2}}, \quad u = \left(\frac{M}{EI}\right)^{1/2} UL \tag{15}$$

for $\eta(\xi, \tau) \propto \exp(i\omega\tau)$, ω being the dimensionless, generally complex frequency. Hence, the system is stable for $\mathscr{I}m(\omega) > 0$, and unstable for $\mathscr{I}m(\omega) < 0$. The results in Table 1(a) confirm the conclusions reached in the previous paragraph.

For the system which is neutrally stable at small u, a divergence occurs at relatively high flow ($u \simeq 2.3$); the occurrence of divergence is quite reasonable, since the system is subjected to an end load which is not a purely tangential follower load.

3.2. Variants of the basic model

Several variants of this model have been devised. They were mainly aimed at generalizing the assumption that fluid facing the inlet is as in Fig. 2(c). Here just one of these variants is presented. A fuller discussion is reserved for the aforementioned full-length paper, since work on this topic is continuing.

Table 1
Eigenfrequencies ω [in (a)] and $\mathscr{I}m(\omega)$ [in (b) and (c)] computed by a two-mode Galerkin solution

(a) Values of ω. Model of Section 3.1 ($\beta = 0.15$)			
$u = 1.0,\ \alpha^* = \sigma = 0$		$u = 1.0,\ \alpha^* = 10^{-3},\ \sigma = 0.05$	
$\alpha = 0$	$\alpha = 0.5$	$\alpha = 0$	$\alpha = 0.5$
3.499 + 0i	3.148 + 0i	3.499 + 0.031i	3.148 + 0.031i
22.14 + 0i	21.77 + 0i	22.14 + 0.268i	21.77 + 0.268i

(b) Values of $\mathscr{I}m(\omega)$. Model of Section 3.2 ($\beta = 0.15,\ \alpha^* = \sigma = 0,\ u = 1$)				
$\alpha = 0.4$ $\bar{\gamma} = 0.8$	$\alpha = 0.4$ $\bar{\gamma} = 0.6$	$\alpha = 0.5$ $\bar{\gamma} = 0.8$	$\alpha = 0.5$ $\bar{\gamma} = 1.0$	$\alpha = 0.5$ $\bar{\gamma} = 1.2$
0.002	−0.001	−0.003	0.0	0.003
−0.002	0.001	0.003	0.0	−0.003

(c) Values of $\mathscr{I}m(\omega)$. Model of Section 3.2 ($\beta = 0.15,\ \alpha^* = 10^{-3},\ \sigma = 0.05,\ u = 1$)				
$\alpha = 0.4$ $\bar{\gamma} = 0.8$	$\alpha = 0.4$ $\bar{\gamma} = 0.6$	$\alpha = 0.5$ $\bar{\gamma} = 0.8$	$\alpha = 0.5$ $\bar{\gamma} = 1.0$	$\alpha = 0.5$ $\bar{\gamma} = 1.2$
0.033	0.030	0.028	0.031	0.034
0.266	0.269	0.271	0.268	0.265

It is presumed that $\bar{p}A = -(1-\alpha)MU^2$, as in Eq. (10a) with $\alpha = v/U$. If $\alpha = \frac{1}{2}$, we obtain $\bar{p}A = -\frac{1}{2}MU^2$, which in fact agrees with Idel'chik (1986): the pressure loss coefficient for a square-cut inlet is 1, i.e. a loss of one velocity head, $\frac{1}{2}\rho U^2$. This is the *steady* pressure loss. However, there will be an additional *unsteady* pressure loss if the pipe undergoes oscillatory motion. Another way of looking at this is that $\bar{p}A = -\frac{1}{2}MU^2$ corresponds to the result obtained via Bernoulli's equation for *steady flow*. However, if flutter is conjectured to arise, the *unsteady form* of the Bernoulli equation ought to have been used instead; the oscillatory motion would result in a further depression of the pressure at the intake, as more fluid kinetic energy would need to be created. Thus, taking $\bar{p}A = -\frac{1}{2}MU^2$ would be very conservative; instead, we retain $\bar{p}A = -(1-\alpha)MU^2$, with $\alpha < \frac{1}{2}$, i.e. $1-\alpha > \frac{1}{2}$.

Furthermore, in the foregoing we had assumed that $T_L = 0$, again a conservative assumption. Taking now $T_L = \bar{\gamma}(1-\alpha)MU^2$, as developed in Appendix A, we can write

$$(T - pA)_L = \overline{T} - \bar{p}A = (1-\alpha)(1+\bar{\gamma})MU^2, \tag{16}$$

the same as Eq. (A.6).

In this variant of the theory, we also assume that the average velocity vector of the fluid entering the pipe inlet is as in Fig. 2(d); hence, $F_z = MU[(\dot{w}_L - Uw'_L) - (-vw'_L)]$. Therefore, with $\alpha = v/U$, we have

$$F_z^* = -MU[\dot{w}_L - (1-\alpha)Uw'_L], \tag{17a}$$

in which, again, $\alpha < \frac{1}{2}$ in view of the foregoing. Since $F_x^* = -\bar{p}A = (1-\alpha)MU^2$, we have

$$F_\zeta^* = -MU\dot{w}_L. \tag{17b}$$

Hence, instead of Eq. (14) we have

$$EI\left(1 + \alpha^*\frac{\partial}{\partial t}\right)\frac{\partial^4 w}{\partial x^4} + [1 - (1-\alpha)(1+\bar{\gamma})]MU^2\frac{\partial^2 w}{\partial x^2} - 2MU\frac{\partial^2 w}{\partial x \partial t} + c\frac{\partial w}{\partial t} + (M + m + M_a)\frac{\partial^2 w}{\partial t^2} + MU\dot{w}\delta(x - L) = 0. \tag{18}$$

Calculating the work done over a period of oscillation, in the absence of dissipation, we obtain

$$\Delta W = -[1 - (1-\alpha)(1+\bar{\gamma})]MU^2\int_0^T \left(\frac{\partial w}{\partial x}\frac{\partial w}{\partial t}\right)\bigg|_L \mathrm{d}t. \tag{19}$$

It is recalled that, in view of the foregoing and Appendix A, $0.7<\bar{\gamma}<1.4$, while $1-\alpha\geqslant 0.5$. It is noted that, if one simply takes $\bar{\gamma}\sim 1$ and $\alpha\sim 0.5$, ΔW is close to zero.

This dynamical behaviour is also illustrated in Table 1(b,c) for $0.6<\bar{\gamma}<1.2$ and $0.4<\alpha<0.5$, with and without dissipation. As $\mathscr{R}e(\omega)$ does not change much with u, only the $\mathscr{I}m(\omega)$ are given.

In the absence of dissipation, it is seen in Table 1(b) that the system is unstable in its second mode if the quantity in square brackets in Eq. (19), $-\bar{\gamma}+\alpha(1+\bar{\gamma})>0$, i.e. in the first and last columns of the table. This makes sense, since $[\overline{(\partial w/\partial x)(\partial w/\partial t)}]_L$ is expected to be negative, at least for low U, and hence $\Delta W>0$. For $-\bar{\gamma}+\alpha(1+\bar{\gamma})<0$, however, the system is unstable in its first mode (second and third columns); again, this makes sense since it corresponds to $[\overline{(\partial w/\partial x)(\partial w/\partial t)}]_L>0$, appropriate for the first-mode oscillation, leading to $\Delta W>0$. Thus, it is interesting that the system "adjusts itself" to extract energy from the fluid *one way or another*. For $-\bar{\gamma}+\alpha(1+\bar{\gamma})=0$ the system is of course neutrally stable.

Nevertheless, it is seen in Table 1(c) that, with $\alpha^*=10^{-3}$ and $\sigma=0.05$, $\mathscr{I}m(\omega)>0$ in all cases, and the system is stable for $u=1$.

Eventually, however, for high enough flow, unless $-\bar{\gamma}+\alpha(1+\bar{\gamma})=0$, the system loses stability by flutter. The critical flow velocities, u_c, are: (i) $u_c=2.2$ in the first mode for $\alpha=0.5$, $\bar{\gamma}=0.8$; (ii) $u_c=7.0$ in the second mode for $\alpha=0.4$, $\bar{\gamma}=0.8$; and (iii) $u_c=6.1$ in the second mode for $\alpha=0.5$, $\bar{\gamma}=1.2$.[4]

3.3. Discussion

It is important to stress that, whether the flow is as in Fig. 1(c) or Fig. 1(d), the effect of the Coriolis forces in the equation of motion is cancelled by the term $-MU(\partial w/\partial t)$ in the boundary condition, insofar as the calculation of ΔW is concerned. Therefore, flutter, in the cases where it does occur, is not related to the Coriolis forces but to nonvanishing fractions of the centrifugal force $MU^2(\partial^2 w/\partial x^2)$, remaining after depressurization and tensioning effects have been accounted for.

It is clear from the foregoing that the dynamics of the system depends intimately on the precise assumptions made regarding the flow-field in the vicinity of the pipe inlet. To advance in this direction, a numerical (CFD) study using ANSYS has been initiated, which should help decide whether the model of Section 3.1 or 3.2, or perhaps another, is closer to the truth, as well as to suggest what the correct values for the parameters in these models should be.

4. Conclusion

In this short paper, the stability of aspirating pipes is reviewed, from (i) the initial study suggesting flutter at infinitesimal flow (Section 2.1), to (ii) the supposed resolution of the problem negating the existence of such flutter (Section 2.2), to (iii) the realization that this supposed resolution is incomplete (Section 2.3) [see Kuiper and Metrikine's (2005) work], and to (iv) the reappraisal of the question (Section 3) with the adaptation of some ideas by Pramila (1992), further developed here.

If one assumes that the flow ingested remains substantially tangential to the mean position of the pipe end in the course of a putative cycle of small oscillation (Section 3.1), it is shown that the system is unconditionally stable.

If the mean velocity vector rotates so as to always be tangential to the pipe inlet during oscillation (Section 3.2), stability is shown to depend on the parameter grouping $-\bar{\gamma}+\alpha(1+\bar{\gamma})$, where $\bar{\gamma}\sim\mathcal{O}(1)$ and $\alpha\sim\mathcal{O}(\frac{1}{2})$. Thus, if $\bar{\gamma}=1$ and $\alpha=\frac{1}{2}$, the system is again unconditionally stable, but otherwise can be unstable if dissipation is not taken into account. With a reasonable amount of dissipation, however, the system is stable in this case also in the range of flow velocities of practical interest. However, since the problem is of fundamental, as well as practical interest, a numerical study has been initiated which will aid in refining and further developing the models presented here.

In terms of the concerns raised with regard to the model of Païdoussis (1999) and expressed as (i) and (ii) in Section 2.3, it has been shown that (i) the Coriolis forces actually do *not* do any net work if the forces at the mouth of the pipe are properly accounted for, and (ii) with or without the use of Bernoulli's equation, the centrifugal forces either totally or nearly vanish; however, it is this last aspect that needs be verified by CFD calculations. Indeed, perhaps this Communication is not quite the final word on this deceptively simple problem.

[4]The calculations for (ii) and (iii) were conducted with eight comparison functions in the Galerkin scheme of solution, because of the high values of u involved.

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Appendix A. Alternative derivations and elaboration of Eqs. (4) and (5)

A.1. Alternative derivations

The arguments leading from Eq. (3) to Eq. (4) effectively say that a relatively large volume of fluid of very small average flow velocity, and with no coherent vectorial sense, changes in a short distance to a smaller volume with coherent mean flow velocity $-U$ aligned with the pipe inlet.

In a sense, this is the converse treatment to that for a short nozzle at the end of a discharging cantilever (Gregory and Païdoussis, 1966a; Païdoussis, 1998), which gives $(T-pA)_L = -MU(U_j - U) = -MU^2(\alpha_j - 1)$, with $\alpha_j = A/A_j > 1$, where subscript j for "jet" denotes quantities at the nozzle exit. It is of interest that Bishop and Fawzy (1976) find the same result for $(T-pA)_L$, even though they determine $p_L A$ via Bernoulli's equation.

In the case of an aspirating pipe, the nozzle is replaced by a funnel leading to the pipe inlet, such that $(T-pA)_L = -MU[-U-(-U_0)]$, U_0 being the average velocity outside in the tangential direction. Thus, if $U_0/U \ll 1$, $(T-pA)_L = MU^2$.

In Païdoussis (1998, 1999) this was obtained by considering a variant of the axial force balance equation, $\partial(T-pA)/\partial x = M(\partial U/\partial t)$, namely

$$\frac{\partial}{\partial x}[T - pA - (MU)U] = 0. \tag{A.1}$$

Considering the funnel idea and integrating from a point outside where $U \simeq 0$ to $x = L$, one again obtains $(T-pA)_L = MU^2$.

The weak point in the foregoing is the "funnel". If we abandon it and use instead Bernoulli's equation, we can write

$$p_L + \frac{1}{2}\rho U^2 = p_u, \tag{A.2}$$

where p_u is the pressure upstream, far enough for the velocity there to be zero. Hence, since in all of this treatment we use pressures relative to the ambient, here equal to p_u for a horizontal system, then $p_L A = -\frac{1}{2}MU^2$ as in Eq. (8). As discussed in Section 3.2, $p_L A$ above is underestimated, because of neglect of unsteady flow effects associated with putative pipe oscillation. Hence, as in Section 3.2, we take

$$p_L A = -(1-\alpha)MU^2, \tag{A.3}$$

where $1 - \alpha > \frac{1}{2}$.

A.2. An expression for $(T - pA)_L$

In Section 2.3, we have presumed that $T_L = 0$, and hence $(T-pA)_L = -(1-\alpha)MU^2$. Actually, there will be a suction on the free-end cross-section of the pipe, so that $T_L > 0$. Assuming the pressure to be somewhere between p_L and the ambient just outside, conservatively taken to be zero, say taking the pressure to be γp_L with p_L as in Eq. (A.3), $0 < \gamma < 1$, we have

$$T_L = -\gamma p_L(A_o - A_i) = -\gamma p_L A_i\left(\frac{A_o - A_i}{A_i}\right) = (1-\alpha)\gamma MU^2\left(\frac{A_o - A_i}{A_i}\right), \tag{A.4}$$

where A_i and A_o are, respectively, the inner and outer cross-sectional areas of the pipe. We may also write Eq. (A.4) as

$$T_L = (1-\alpha)\gamma MU^2\frac{\rho}{\rho_s}\left(\frac{1}{\beta_i} - 1\right) \equiv (1-\alpha)f\gamma MU^2 \equiv (1-\alpha)\overline{\gamma}MU^2, \tag{A.5}$$

where ρ_s is the density of the pipe material, and $\beta_i = M/(M+m)$. The reason for introducing the form involving β_i is that in most of the published literature the parameter β_i is used (mostly without the subscript i).

For a fairly thick elastomer pipe conveying water, typical of those used in the experiments ($\beta_i = 0.24$, ratio of outer/inner diameter $D_o/D_i = 1.95$, $\rho_s = 1.11\rho$), we obtain $f = 2.85$; for a thicker pipe ($\beta_i = 0.15$, $D_o/D_i = 2.44$), $f = 4.95$, while for a thinner one ($\beta_i = 0.52$, $D_o/D_i = 1.36$), $f = 0.84$. For an air-conveying elastomer pipe ($\beta_i \simeq 10^{-3}$), $f \simeq 1.0$. For a steel pipe conveying water ($\beta_i = 0.15$, $D_o/D_i = 1.3$, $\rho_s = 7.8\rho$), $f = 0.72$. Thus, for relatively thick pipes $f > 2$, while for relatively thin ones $f \simeq 0.8$ is a more representative value. Now, generally, for the thicker pipes γ will be lower (say, $\gamma = 0.5$) than for the thinner ones where the pressure on the pipe face should be closer to the pressure in the pipe mouth (hence $\gamma \simeq 1$). Consequently, putting all the foregoing together, we obtain $0.7 < \bar{\gamma} \equiv f\gamma < 1.4$ approximately, or $\bar{\gamma} \sim \mathcal{O}(1)$. The point is that T_L is not negligible, and $(T - pA)_L \simeq MU^2$ is not too grossly unreasonable, after all!

Hence, we take T_L as in Eq. (A.5), and therefore

$$(T - pA)_L = (1 - \alpha)(1 + \bar{\gamma})MU^2, \tag{A.6}$$

where it is understood that $\alpha \sim \mathcal{O}(\frac{1}{2})$ but larger than 0.5 and $\bar{\gamma} \sim \mathcal{O}(1)$. The fact that $(1 - \alpha)(1 + \bar{\gamma}) \sim \mathcal{O}(1)$ is important, because the tensioning-pressurization term associated with Eq. (A.6) in the equation of motion tends to cancel out the centrifugal force, substantially if not completely.

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